Conference

'Analysis of PDEs: unique continuation, stabilization, control and dispersive properties'

Institut Henri-Poincaré, November 5–9, 2018

Organisers: Jean-Marc Delort, Jérôme Le Rousseau

Scientific Committee: Patrick Gérard, Otared Kavian, Maciej Zworski

PROGRAM

Monday, November 5

Location: Institut Henri-Poincaré, Amphithéatre Hermite

8:50–9:00: Introduction

9:00–10:00: **Claude Zuily** (Université Paris-Sud, Orsay).
Cauchy theory for the gravity water waves system in an analytic framework.

10:00–10:20: *Coffee Break*

10:20–11:20: **Johannes Sjöstrand** (Université de Bourgogne, Dijon).
Fourier integral operators and Bergman projections for weighted spaces of holomorphic functions.

11:20–12:20: **Mourad Bellassoued** (ENIT, Tunis).
A Borg-Levinson theorem for magnetic Schrödinger operators on a Riemannian manifold.

12:20–14:00: *Lunch*

14:00–15:00: **Enrique Zuazua**(DeustoTech-Bilbao, UAM-Madrid, and LJLL, Sorbonne Université).
Control of a population dynamics model with age structuring and diffusion.

15:00–16:00: **Emmanuelle Crépeau** (Université Grenoble Alpes).
Some controllability results for the Korteweg-de Vries equation.

16:00–16:20: *Coffee Break*

16:20–17:20: **Belhassen Dehman** (Université de Tunis - El Manar).
Observability for the wave equation with non smooth metric.

Tuesday, November 6

Location: Institut Henri-Poincaré, Amphithéatre Hermite

9:00–10:00: **Jean-Michel Coron** (Sorbonne Université, Paris).
Few methods to stabilize rapidly various control systems.

10:00–10:20: *Coffee Break*

10:20–11:20: **Assia Benabdallah** (Aix-Marseille Université).
Control of Parabolic Equations.

11:20–12:20: **Vahagn Nersesyan** (Université de Versailles Saint-Quentin).
On some relations between ergodicity and controllability for stochastic systems.

12:20–14:00: *Lunch*

14:00–15:00: **Hajer Bahouri** (Université Paris-Est Créteil).
Blow up mechanism for quasilinear wave equations.

15:00–16:00: **Thomas Duyckaerts** (Université Paris 13).
Semilinear wave equations outside a ball.

16:00–16:20: *Coffee Break*

16:20–17:20: **Yvan Martel** (École polytechnique, Palaiseau).
Construction of minimal mass blow up solutions for the mass critical gKdV and mBO equations.

Wednesday, November 7

Location: Université de Versailles Saint-Quentin

9:00–10:00: **Marius Tucsnak** (Université de Bordeaux).
The piston problem: well-posedness, stability and controllability.

10:00–10:20: *Coffee Break*

10:20–11:20: **Lucie Baudouin** (CNRS, Université de Toulouse).
Reconstruction procedure of coefficient in the wave equation.

11:20–12:20: **Moez Khenissi** (University of Sousse).
On the damped Schrödinger equation in unbounded domains.

12:20–14:20: *Conference lunch*

Thursday, November 8

Location: Institut Henri-Poincaré, Amphithéatre Hermite

9:00–10:00: **Jean-Claude Saut** (Université Paris-Saclay, Orsay).
The Cauchy problem for the fractionary Kadomtsev-Petviashvili equations.

10:00–10:20: *Coffee Break*

10:20–11:20: **Karine Beauchard** (ENS Rennes).
Unexpected quadratic behaviors for the small-time local null controllability of scalar-input PDEs.

11:20–12:20: **Jared Wunsch** (Northwestern University, USA).
Diffraction and propagation of semiclassical singularities for Schrödinger operators with conormal potentials.

12:20–14:00: *Lunch*

14:00–15:00: **Julie Valein** (Université de Lorraine, Nancy).
Stabilization of nonlinear KdV equation with time-delay feedback.

15:00–16:00: **Valeria Banica** (Sorbonne Université, Paris).
Evolution of polygonal lines by the binormal flow.

16:00–16:20: *Coffee Break*

16:20–17:20: **Luis Vega** (BCAM, Bilbao).
The Binormal Flow, the Talbot effect, and non-circular jets.

17:20–18:20: **Nicolas Burq** (Université Paris-Sud, Orsay).
Sharp resolvent and time decay estimates for dispersive equations on asymptotically Euclidean backgrounds.

Friday, November 9

Location: Institut Henri-Poincaré, Amphithéatre Hermite

9:00–10:00: **Nicolas Lerner** (Sorbonne Université, Paris).
Unique continuation through transversal characteristic hypersurfaces.

10:00–10:20: *Coffee Break*

10:20–11:20: **David Dos Santos Ferreira** (Université de Lorraine, Nancy).
On the linearized anisotropic Calderón problem.

11:20–12:20: **Matthieu Léautaud** (École polytechnique, Palaiseau).
Quantitative Tataru-Robbiano-Zuily-Hörmander theorem and applications.

12:20–14:00: *Lunch*

14:00–15:00: **Gilles Lebeau** (Université Côte d'Azur - Nice).
Inégalités de Carleman.

Abstracts

Hajer Bahouri (Université Paris-Est Créteil).
Blow up mechanism for quasilinear wave equations.

Abstract. In this lecture, we consider the hyperbolic vanishing mean curvature flow of surfaces in $\mathbb{R}^8$ asymptotic at infinity to Simons cone:

$$C_4 = \Big\{ X = (x_1, \cdots, x_8) \in \mathbb{R}^8 \, , x_1^2 + \cdots + x_4^2 = x_5^2 + \cdots + x_8^2 \Big\} \, ,$$

and show that the flow admits finite time blow up solutions $(\Gamma(t))_{0<t\leq T}$ that blow up by concentration of the stationary profile: there exists a smooth minimal surface M asymptotic at infinity to Simons cone such that

$$\Gamma(t) \sim t^{\nu+1} \, M \, , \text{ as } t \to 0 \, ,$$

uniformly on compact sets, where ν is an arbitrary large positive number.

This issue amounts to investigate the singularity formation for a second order quasilinear wave equation. Our constructive approach consists in proving the existence of finite time blow up solutions of this hyperbolic equation under the form $u(t,x) \sim t^{\nu+1} Q\Big(\frac{x}{t^{\nu+1}}\Big)$, where Q is a stationary solution.

Valeria Banica (Sorbonne Université, Paris).
Evolution of polygonal lines by the binormal flow.

Abstract. The binormal flow is a 3-D curve evolution equation that models the dynamics of filament vortices in 3-D fluids. The first part of this talk will be dedicated to the presentation of the classical connection established between the binormal flow and the 1-D cubic Schrödinger equation. We shall see in particular that the 1-D cubic Schrödinger equation with initial data a Dirac mass is related to the formation in finite time of a corner by the binormal flow. As a first result we construct solutions of the 1-D cubic Schrödinger equation in link with initial data a sum of several Dirac masses. Then we shall construct and describe a new class of singular solutions of the binormal flow, that in particular we continue after the time of singularity formation. This is a joint work with Luis Vega.

Lucie Baudouin (CNRS, Université de Toulouse).
Reconstruction procedure of coefficient in the wave equation.

Abstract. This talk will aim at presenting the work developed over the last few years in collaboration with Maya de Buhan, Sylvain Ervedoza and Axel Osses regarding coefficient inverse problems for the wave equation.

The scientific goal we pursue is to propose new and efficient reconstruction procedures for unknown coefficients of the wave equation from the knowledge of the flux of the solution

(Neumann boundary data) on a part of the domain's boundary. So far, we studied the cases of the determination of the potential $q(x)$ (a zeroth order term coefficient) or of the speed $a(x)$ (main coefficient) in the wave equation with given Dirichlet boundary data. Several uniqueness and stability results are available in the literature about these inverse problems and in particular from Imanuvilov and Yamamoto in both cases. Our approach and main interest is to use, in the reconstruction procedure we propose, the same technical tool that proves the stability result, namely some appropriate Carleman estimates.

We will present the two globally convergent reconstruction algorithms for the potential and the speed coefficients. The design and convergence of the algorithms are based on global Carleman estimates for the wave operator. However, the numerical implementation of our strategy presents several challenges, most of which we will address here. Finally, we will give some illustrative numerical simulations.

Karine Beauchard (ENS Rennes).
Unexpected quadratic behaviors for the small-time local null controllability of scalar-input PDEs.

ABSTRACT. We consider scalar-input control systems in the vicinity of an equilibrium, at which the linearized systems are not controllable. For finite dimensional control systems, we recently classified the possible quadratic behaviors. Quadratic terms introduce coercive drifts in the dynamics, quantified by integer negative Sobolev norms, which are linked to Lie brackets and which prevent smooth small-time local controllability for the full nonlinear system. In the context of nonlinear parabolic equations, we prove that the same obstructions persist. More importantly, we prove that two new behaviors occur, which are impossible in finite dimension. First, there exists a continuous family of quadratic obstructions quantified by fractional negative Sobolev norms or by weighted variations of them. Second, and more strikingly, small-time local null controllability can sometimes be recovered from the quadratic expansion. We also construct a system for which an infinite number of directions are recovered using a quadratic expansion. As in the finite dimensional case, the relation between the regularity of the controls and the strength of the possible quadratic obstructions plays a key role in our analysis.

This is a joint work with Frederic Marbach

Mourad Bellassoued (ENIT, Tunis).
A Borg-Levinson theorem for magnetic Schrödinger operators on a Riemannian manifold.

ABSTRACT. This article is concerned with uniqueness and stability issues for the inverse spectral problem of recovering the magnetic field and the electric potential in a Riemannian manifold from some asymptotic knowledge of the boundary spectral data of the corresponding Schrödinger operator under Dirichlet boundary conditions. The spectral data consist of some asymptotic knowledge of a subset of eigenvalues and Neumann traces of the associated eigenfunctions of the magnetic Laplacian. We also address the same question for Schrödinger operators under Neumann boundary conditions, in which case we measure the Dirichlet traces of eigenfunctions. In our results we characterize the

uniqueness of the magnetic field from a rate of growth of the eigenvalues, combined with suitable asymptotic properties of boundary observation of eigenfunctions, of the associated magnetic Schrödinger operator. To our best knowledge this is the first result proving uniqueness from such general asymptotic behavior of boundary spectral data.

Assia Benabdallah (Aix-Marseille Université).
Control of Parabolic Equations.

ABSTRACT. One of the main goal in control theory is to drive the state of the system to a given configuration using a control that acts through a source term located inside the domain or on the boundary.

The reference works for the control of linear parabolic problems are due to H.O. Fattorini and D.L. Russell, in the 70's for the one dimensional case and to A.V. Fursikov, O.Yu. Imanuvilov, on one side and G. Lebeau, L. Robbiano, on the other side both in the 90's for the multi-dimensional case. They established null-controllability of heat equations with distributed or boundary controls at any time $T > 0$ and for any control domain.

Recently, several studies of control of parabolic equations have revealed new behaviors as, for instance, minimal time of control, geometrical dependence on the location of the control. The aim of this talk is to try unify these examples. It is based on joint works with F. Ammar-Khodja, F. Boyer, M. González-Burgos, M. Morancey, and L. de Teresa.

Nicolas Burq (Université Paris-Sud, Orsay).
Sharp resolvent and time decay estimates for dispersive equations on asymptotically Euclidean backgrounds.

ABSTRACT. In this talk, I will present a very robust method for proving sharp time decay estimates for the most classical three models of dispersive Partial Differential Equations, the wave, Klein-Gordon and Schrödinger equations, on curved asymptotically flat geometries, showing under very general assumptions the exact same decay as for the Euclidean case. I will also show how to extend these decay properties to the case of boundary value problems. This is based on a work with J.M. Bouclet.

Jean-Michel Coron (Sorbonne Université, Paris).
Few methods to stabilize rapidly various control systems.

ABSTRACT. We present various methods for the rapid stabilization of control systems modeled by means of ordinary differential equations or partial differential equations. These methods are explained on a slider moving on a plane, 1-D linear hyperbolic systems, 1-D linear parabolic equations and the nonlinear viscous Burgers equation.

Emmanuelle Crépeau (Université Grenoble Alpes).
Some controllability results for the Korteweg-de Vries equation.

ABSTRACT. In this talk, we will present some results on the boundary controllability of the Korteweg-de Vries equation. We will consider first the case of control on a bounded domain. In that case the controllability of the linearized equation depends on the length of the domain. We will prove the local exact controllability of the nonlinear problem. Then we will study the equation on a star shaped network with control on external nodes.

Belhassen Dehman (Université de Tunis - El Manar).
Observability for the wave equation with non smooth metric.

ABSTRACT. The property of observability for the wave equation is a central question since it is linked to controllability. It has been intensively studied, mainly in a smooth framework (smooth metric and geometry). In this lecture, we will present some results on observability/control for the wave equation with rough coefficients. In particular, we will prove that the observability property is stable under small Lipschitz perturbations of the metric. In addition, we will discuss the sensitivity to these perturbations of the HUM control process.

David Dos Santos Ferreira (Université de Lorraine, Nancy).
On the linearized anisotropic Calderón problem.

ABSTRACT. The anisotropic Calderón problem is the inverse problem consisting in determining a metric on a compact Riemannian manifold with boundary from the Dirichlet-to-Neuman map. The resolution of the problem in a conformal class follows from a similar inverse problem on the Schrödinger equation and remains an open question in dimensions higher than 3. In previous works, we could solve this inverse problem under structural assumptions on the known metric (namely that it is conformal to a warped product with an Euclidean factor) and additional geometric assumptions on the transversal manifold. The proof of uniqueness relies on the high frequency limit in a Green identity involving pairs of complex geometrical optics solutions to the Schrödinger equation. This talk will be concerned with a description of the resolution of this nonlinear inverse problem under strong assumptions on the metric and our attempts to remove the additional transversal assumptions on the geometry by refraining from passing to the limit in the linearised problem. Unfortunately, this path only leads to partial results on the linearised problem for the time being, that is recovery of singularities of the potential in the transversal variables. This a joint work with Yaroslav Kurylev, Matti Lassas, Tony Liimatainen and Mikko Salo.

Thomas Duyckaerts (Université Paris 13).
Semilinear wave equations outside a ball.

Abstract. Consider radial solutions of the wave equation outside a ball in 3 space dimensions with Dirichlet boundary conditions, and a focusing superquintic nonlinearity. This equation has a countable family of stationary solutions. I will prove that any global solution converges, up to a dispersive term, to one of the stationary solutions. This is in sharp contrast with the case without obstacle, for which there is no stationary solution and any bounded global solution scatters to a linear solution.

This is a joint work with Jianwei Yang.

Moez Khenissi (University of Sousse).
On the damped Schrödinger equation in unbounded domains.

Abstract. In this talk, we survey some of our works on the damped Schrödinger equations in unbounded domains. We prove in particular, under the geometric control condition, the smoothing effect proprieties and the local energy decay.

Matthieu Léautaud (École polytechnique, Palaiseau).
Quantitative Tataru-Robbiano-Zuily-Hörmander theorem and applications.

Abstract. In this talk, after having explained its interest for the wave equation, we shall recall the celebrated Tataru-Robbiano-Zuily-Hörmander theorem concerning unique continuation for differential operators with partially analytic coefficients. We shall then give local and global (optimal) inequalities quantifying this theorem. Finally, we shall present several applications of such inequalities to the approximate and exact controllability of waves, and to hypoelliptic equations. This is based on joint works with Camille Laurent.

Gilles Lebeau (Université Côte d'Azur - Nice).
Inégalités de Carleman.

Nicolas Lerner (Sorbonne Université, Paris).
Unique continuation through transversal characteristic hypersurfaces.

Abstract. We prove a unique continuation result for an ill-posed characteristic problem. A model problem of this type occurs in A.D. Ionescu & S. Klainerman article (Theorem 1.1 in [MR2470908]) and we extend their model-result using only geometric assumptions. The main tools are Carleman estimates and Hörmander's pseudo-convexity conditions.

Yvan Martel (École polytechnique, Palaiseau).
Construction of minimal mass blow up solutions for the mass critical gKdV and mBO equations.

ABSTRACT. This talk will present the construction of minimal mass blow up solutions for the critical generalized Korteweg-de Vries equation (with Merle-Raphaël and Combet) and the modified Benjamin-Ono equation (with Pilod).

[1] V. COMBET AND Y. MARTEL, *Sharp asymptotics for the minimal mass blow up solution of critical gKdV equation*, Bulletin des Sciences Mathématiques, Elsevier, 2017, 141 (2), pp.20–103.
[2] V. COMBET AND Y. MARTEL, *Construction of multi-bubble solutions for the critical gKdV equation,* SIAM Journal on Mathematical Analysis, Society for Industrial and Applied Mathematics, **50** (2018), 3715–3790.
[3]C. E. KENIG, Y. MARTEL, L. ROBBIANO, *Local well-posedness and blow up in the energy space for a class of L^2 critical dispersion generalized Benjamin-Ono equations*, Ann. Inst. Henri Poincaré, Anal. Non Linéaire **28**, No. 6, 853–887 (2011).
[4] Y. MARTEL, F. MERLE AND P. RAPHAËL, *Blow-up for the critical generalized Korteweg-de Vries equation I: dynamics near the soliton,* Acta Math., **212** (2014), 59–140.
[5] Y. MARTEL, F. MERLE AND P. RAPHAËL, *Blow-up for the critical generalized Korteweg-de Vries equation II: minimal mass dynamics,* J. Eur. Math. Soc., **17** (2015), 1855–1925.
[6] Y. MARTEL AND D. PILOD, *Construction of a minimal mass blow up solution of the modified Benjamin–Ono equation,* Mathematische Annalen, Springer Verlag, 2017, 369 (1-2), pp.153–245.
[7] F. MERLE, *Construction of solutions with exactly k blow-up points for the Schrödinger equation with critical nonlinearity,* Comm. Math. Phys, **129** (1990), 223–240.
[8] P. RAPHAËL AND J. SZEFTEL, *Existence and uniqueness of minimal blow-up solutions to an inhomogeneous mass critical NLS,* J. Amer. Math. Soc., **24** (2011), 471–546.

Vahagn Nersesyan (Université de Versailles Saint-Quentin).
On some relations between ergodicity and controllability for stochastic systems.

ABSTRACT. In this talk, we will discuss some recent results on the long-time behaviour of solutions of PDEs perturbed by a random force. We will show that under some generic controllability hypotheses (satisfied for Navier-Stokes and Ginzburg-Landau equations), there is a unique invariant measure which attracts exponentially all the solutions. This is a joint work with S. Kuksin and A. Shirikyan.

Jean-Claude Saut (Université Paris-Saclay, Orsay).
The Cauchy problem for the fractionary Kadomtsev-Petviashvili equations.

ABSTRACT. Following the formal approach of Kadomtsev and Petviashvili (1970) one can derive a weakly transverse extension of any one-dimensional nonlinear dispersive equation. When applied to the Korteweg-de Vries (KdV) one obtains the classical KP-I and KP-II equations. Starting from fractionary KdV equations one obtains the fractionary KP equations (fKP). They include in particular the KP version of the Benjamin-Ono (BO) equation or of the Intermediate Long Wave (ILW) equation which are of special interest. We will review recent well-posedness and ill-posedness results on the Cauchy problem associated to fKP equations and on some more qualitative aspects. This is based on joint works with Felipe Linares and Didier Pilod.

Johannes Sjöstrand (Université de Bourgogne, Dijon).
Fourier integral operators and Bergman projections for weighted spaces of holomorphic functions.

ABSTRACT. In the first part (based on a joint work with L. Coburn and M. Hitrik) we discuss positivity and boundedness of metaplectic operators, acting on spaces of entire functions with quadratic exponential weights (Bargmann spaces) and the relations with Toeplitz and Weyl quantizations, including an interesting example of C. Berger and L. Coburn.

In the second part (based on a joint work with O. Rouby and S. Vũ Ngọc) we consider spaces with real-analytic exponential weights with strictly plurisubharmonic exponents, in the semi-classical case. We show that analytic pseudodifferential operators and certain "Bergman-Bargmann" quantizations with analytic symbols agree up to exponentially small errors. This includes asymptotic Bergman projections. Exact Bergman projections are treated in the case of functions on $\mathbf{C}^n$ and sections of high powers of complex line bundles.

Marius Tucsnak (Université de Bordeaux).
The piston problem: well-posedness, stability and controllability.

ABSTRACT. We consider the coupled PDEs/ODEs system modelling the motion of a solid in a viscous heat conducting gas. We first develop a systematic approach to obtain local well-posedness and asymptotic stability results. We next show that in the one dimensional case (the piston problem) some of our results are global. We also discuss the so called "adiabatic piston" problem, which is still of big interest in statistical physics. Finally, we show that for a simplified problem we obtain finite time controllability.

Julie Valein (Université de Lorraine, Nancy).
Stabilization of nonlinear KdV equation with time-delay feedback.

ABSTRACT. This talk concerns the nonlinear Korteweg-de Vries equation with boundary time-delay feedback. Under appropriate assumption on the coefficients of the feedbacks (delayed or not), we first prove that this nonlinear infinite dimensional system is well-posed for small initial data. The main results of our study are two theorems stating the exponential stability of the nonlinear time delay system. Two different methods are employed: a Lyapunov functional approach (allowing to have an estimation on the decay rate, but with a restrictive assumption on the length of the spatial domain of the KdV equation) and an observability inequality approach, with a contradiction argument (for any non critical lengths but without estimation on the decay rate). Some numerical simulations are given to illustrate the results.

Luis Vega (BCAM, Bilbao).
The Binormal Flow, the Talbot effect, and non-circular jets.

ABSTRACT. We shall propose the Binormal Flow as a possible toy model for turbulence, in particular due to its striking similarity with the dynamics exhibited by non-circular jets. This similarity involves the existence of some type of non-linear Talbot effect. Theoretical and numerical arguments about the transfer of energy will be also given. This a joint work with V. Banica and F. de la Hoz.

Jared Wunsch (Northwestern University, USA).
Diffraction and propagation of semiclassical singularities for Schrödinger operators with conormal potentials.

ABSTRACT. Let V be a real potential with a conormal singularity across a hypersurface Y. We consider the semiclassical Schrödinger equation

$$(h^2\Delta + V - E)u = 0$$

and consider the effect of the singularity of V on the propagation of singularities. There turns out to be a 'diffractive' reflection of singularities from Y, with a strength (measured in powers of h) that decreases as V becomes more regular. This diffractive propagation has interesting consequences for distribution of resonances in $O(h\log(1/h))$ neighborhoods of the real axis.

The propagation results generalize some old one-dimensional results of Berry as well as some more recent results of De Hoop–Uhlmann–Vasy on the wave equation. This is joint work with Oran Gannot.

Enrique Zuazua(DeustoTech-Bilbao, UAM-Madrid, and LJLL, Sorbonne Université).
Control of a population dynamics model with age structuring and diffusion.

ABSTRACT. This lecture is devoted to present recent joint work in collaboration with D. Maity and M. Tucsnak (Univ. Bordeaux) on a linear system in population dynamics involving age structuring and spatial diffusion (of Lotka-McKendrick type). The control is localized in space and age. We prove that the whole population can be steered to zero in a uniform time, without, as in the existing literature, excluding some interval of low ages. And we do it in a sharp time. We also show that the system can be steered between two positive steady states by controls preserving the positivity of the state trajectory, something that plays a key role in applications.

Claude Zuily (Université Paris-Sud, Orsay).
Cauchy theory for the gravity water waves system in an analytic framework.

ABSTRACT. For the system of gravity water waves in a domain with a free upper boudary and a flat bottom, with holomorphic data of size ε and a data of Sobolev regularity on the bottom, we prove the existence of a solution on a time interval of size C/ε, whose the domain of holomorphy shrinks for large time. Joint work with Thomas Alazard and Nicolas Burq.